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NUMERICAL INVESTIGATION OF THE ASCENSION OF GAS BUBBLES IN AN INFINITE LIQUID MEDIUM

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Abstract. *This work reports the results of a numerical investigation of the flow around spherical gas bubbles with terminal rising speed, as well as the determination of some dimensionless parameters (e.g., Reynolds numbers, drag coefficients, etc) that characterize this type of flow. To obtain the solution of the problem, the authors used a computational program that solves the non-linear Navier-Stokes Partial Differential Equations (PDE's) and the continuity equation (pressure equation) using the Finite Element Method. Simultaneously, the stream function equation was solved to visualize the flow around the bubble. Finally, the numerical results for the gas bubbles were compared with the data previously obtained by the authors for inviscid bubbles and for rigid spheres and with other results of the specific literature.*

Keywords: *Gas Bubble, two-phase flow, drag coefficient*

1. INTRODUCTION

The flow involving gaseous mixtures without thermal change is governed by the fundamental principles of the mass conservation and Newton's Second law, modeled mathematically by the Navier-Stokes and continuity system of nonlinear partial differential equations (PDE's).

Several works investigating gas bubbles movement in a liquid medium were published in the last decades. The first publications on this subject used the analytic and experimental techniques, but the numerical works prevailed after computer development.

In de Oliver & Chung (1987) work, a combination of the technique of series-truncation with the finite element method was used to simulate the steady flow inside and around liquid droplets for low Reynolds numbers. The obtained results showed good agreement with experimental data presented in the literature.

Karamanev (1994) proposed a semianalytical equation for the drag coefficient of single gas bubble rising in quiescent liquid medium. This equation was obtained assuming that the internal bubble recirculation has no effect on the rising velocity of the bubble and, therefore, the drag coefficient of the rising bubble equals that of a rising light solid particle (solid bubble). Karamanev obtained a good agreement among the speed given by his semianalytical equation and the experimental data published in the literature.

Silveira Neto (1998) performed a numerical investigation of two-dimensional simulations of single bubble transport. His work allowed the visualization of the internal flow in the bubble, showing the existence of counter-rotating cells inside the bubble, created, according to the author, by viscous friction since no particle can cross the interface of the bubble.

This work studied the ascending displacement of gaseous bubbles immersed in an infinite liquid medium. The Galerking Finite Element Method is used to solve the Navier–Stokes and continuity equations' system. The numerical results for velocity components and pressure allowed the calculation of the bubble's drag coefficient.

The numerical results for the bubbles were compared with data obtained by the authors for inviscid bubbles and for rigid spheres and with other results in the specific literature.

2. PHYSICAL-MATHEMATICAL MODEL

In the physical model used to analyze the flow inside and around a gas bubble, it is admitted as a simplifying hypotheses, continuous media, Newtonian fluids, constant physical properties, steady state regime and axi-symmetric geometry. The mathematical model used in this work is given by the non-linear Navier-Stokes and the continuity equations, in spherical coordinates (r, θ), presented below:

- Continuity equation:

$$\frac{1}{r}\frac{\partial\left(r^2 u\right)}{\partial r}+\frac{1}{\operatorname{sen}(\theta)}\frac{\partial\left(v\operatorname{sen}(\theta)\right)}{\partial\theta}=0 \qquad (1)$$

- Momentum equations:

component - r

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial\theta}-\frac{v^2}{r}\right)=-\frac{\partial p}{\partial r}+ \mu\left[\nabla^2 u-\frac{2u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta}-\frac{2}{r^2}v\cot g(\theta)\right]+\rho g_r \qquad (2)$$

component - θ

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}-\frac{vu}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\nabla^2 v+\frac{2}{r^2}\frac{\partial u}{\partial \theta}-\frac{v}{r^2\,\mathrm{sen}(\theta)}\right]+\rho g_\theta \qquad (3)$$

where u is the speed in the radial direction, v is the speed in the θ direction, p is the pressure, g_r is the acceleration due to gravity in the r direction, g_θ is the acceleration due to gravity in the θ direction, μ is the molecular viscosity and ρ is the density of the fluid. $\nabla^2\bullet$ is the Laplacian of the variable $\bullet$.

These equations were implemented in the *PDEase2D* program (Reference Manual, 1996) and solved by the Galerkin Finite Element Method.

The drag force was calculated as:

$$F_A = F_p + F_{vn} + F_{vt} \qquad (4)$$

where the component of the pressure force in the flow direction is given by:

$$F_p = \int_S p\cos(\theta)\, dS \qquad (5)$$

The components of viscous forces in the flow direction due to normal and tangential viscous tensions are given respectively by the Eqs. (6) and (7).

$$F_{vn} = \int_S -\tau_{rr}\cos(\theta)\ \partial S \qquad (6)$$

$$F_{vt} = \int_S \tau_{r\theta}\cos\left(\frac{\pi}{2}-\theta\right)\ \partial S \qquad (7)$$

where:

$$\tau_{rr} = \mu_l\, 2\frac{\partial u}{\partial r} \qquad (8)$$

and

$$\tau_{r\theta} = \mu_l \left(r \frac{\partial \left(\frac{v}{r} \right)}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right) \tag{9}$$

The drag coefficients was calculated by:

$$C_A = \frac{F_A}{\frac{1}{2} \rho_L \, Ve^2 \, A} \tag{10}$$

where ρ_L is the liquid density, A is the bubble projected area and Ve is uniform flow speed.

In this model, the pressure is calculated by a Poisson equation obtained from the momentum and continuity equations. This Poisson equation is presented below:

$$\begin{aligned} &\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \, \mathrm{sen}(\theta)} \frac{\partial}{\partial \theta} \left(\mathrm{sen}(\theta) \frac{\partial p}{\partial \theta} \right) = \\ &- \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \rho \left(u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} \right) \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \rho g_r \right) - \\ &- \frac{1}{r \, \mathrm{sen}(\theta)} \frac{\partial}{\partial \theta} \left[\mathrm{sen}(\theta) \rho \left(u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} - \frac{v u}{r} \right) \right] + \frac{1}{r \, \mathrm{sen}(\theta)} \frac{\partial}{\partial \theta} \left(\mathrm{sen}(\theta) \rho g_\theta \right) \end{aligned} \tag{11}$$

3. BOUNDARY CONDITIONS

The domain adopted takes into account the physics of the problem, where the gaseous bubble is immersed in a uniform flow with speed *Ve*.

Two rectangular regions in spherical coordinates represent the physical domain in cylindrical coordinates.

The domain boundary surfaces will be identified by a *Bi* symbol as shown in Fig. 1.

The external dimension of the domain was fixed in r = 80 R_G, where R_G is the bubble's radium, following Cuenot et al. (1997) whom verified that this flow boundary condition doesn't disturb the external flow solution.

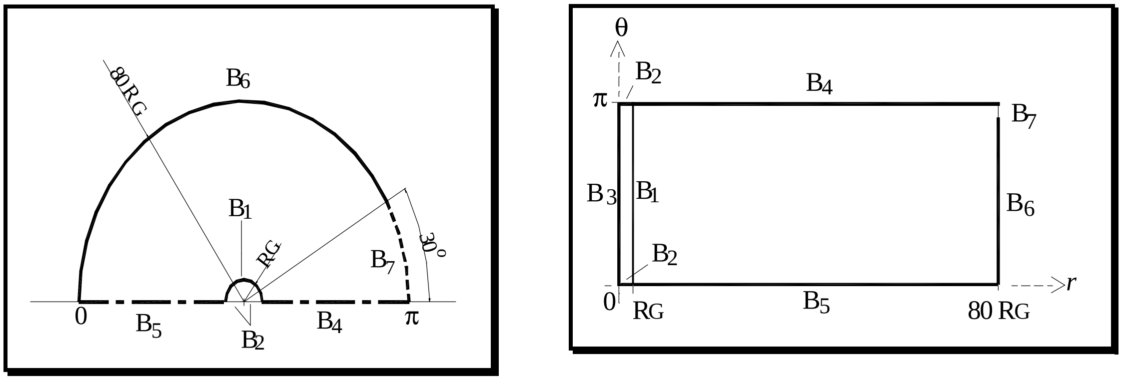

Figure 1 - Flow Domain in cylindrical and spherical coordinates.

In *B2*, *B4* and *B5* symmetry boundary conditions are prescribed:

$$\frac{\partial u}{\partial \theta} = 0, \quad v = 0 \quad e \quad \frac{\partial p}{\partial \theta} = 0 \tag{12}$$

At the boundary *B6*, the conditions applied are presented below:

$$u = -Ve\cos(\theta), \quad v = Ve\cos(\theta) \quad e \quad \frac{\partial p}{\partial r} = \rho\, g_r \tag{13}$$

The boundary conditions for *u* and *v* determine a uniform profile for the flow in the external region, creating a field of uniform flow field where the gas bubble is immersed.

As shown in Batchelor (1967) the rigid sphere and gaseous bubble wakes don't exceed $\phi = 30^o$ at $r = 80\, R_G$. At the outlet B_7 surface, free boundary conditions are imposed for *u* and *v*, and prescribed value for *p*. These boundary conditions are:

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial v}{\partial r} = 0 \quad e \quad p = -\rho_l\, g\, 80 R_G \cos(\theta) \tag{14}$$

Due to impermeability at the bubble surface, a $u = 0$ restriction is imposed at the B_1 surface.

4. RESULTS AND CONCLUSIONS

The numerical results were tested for grid refinement as to obtain a grid independent solution.

Fig. 3 exhibits a comparison among the drag coefficient for the gas bubbles of this work and the results presented by Pantaleão (1999) for the drag coefficient of the inviscid bubbles and for rigid spheres. In the same graph were plotted the results obtained by Magnaudet (1995) relative to inviscid bubbles, and the data of two correlations regarding rigid sphere and gas bubbles presented respectively by Karamanev (1994) and Moore (1963).

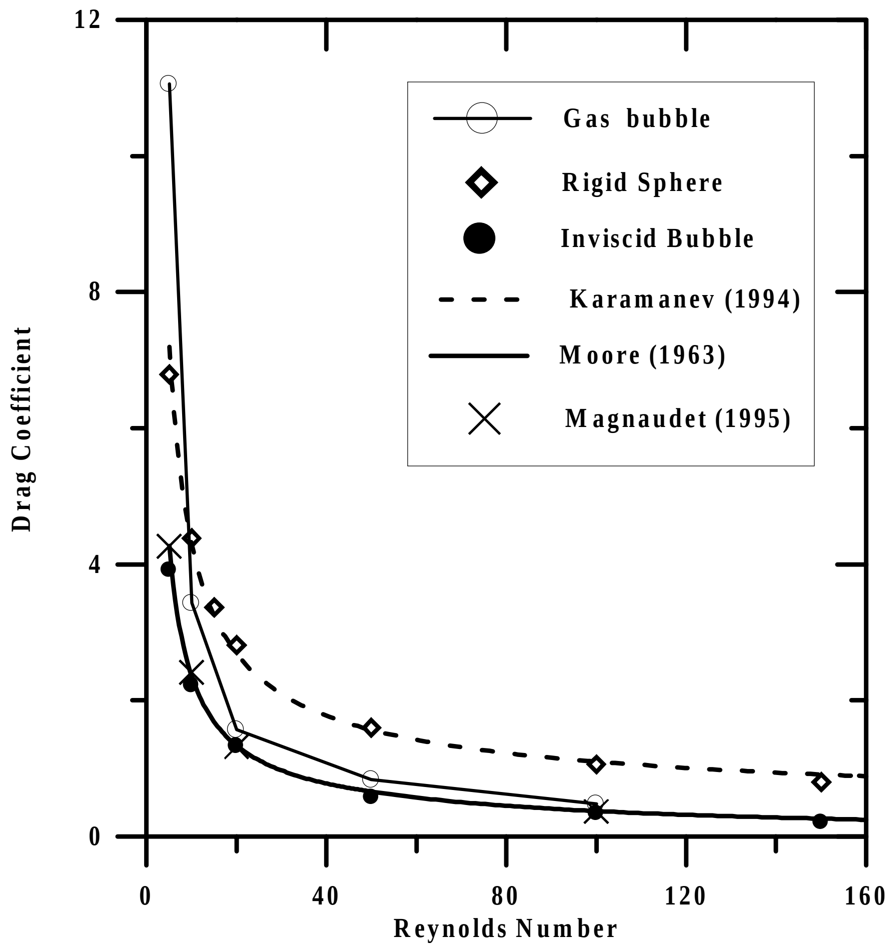

Figure 3 - Gas bubble drag coefficient.

It can be seen that the numerical results obtained for the gas bubbles for Reynolds numbers between 5 and 100, follow the geometric form of the curves suggested by the data in the literature for rigid spheres and for inviscid bubbles. However, the results obtained for the gas bubbles are shifted down when compared with the results obtained for rigid spheres. That was expected, since the sliding of the internal end external fluids at the bubble's surface reduces the drag force. These results are slightly higher than those for the inviscid bubble. As the air viscosity is much smaller than the water viscosity, the results for the drag coefficient of the gaseous bubble are very close to the ones for the inviscid bubble.

However, for Reynolds numbers smaller than 5, it is observed that the drag coefficient of the gas bubble is greater than the drag coefficient of the rigid sphere.

For Reynolds numbers greater than 100, instability is observed in the numerical solution due to the lack of an *upwind* scheme in the *PDEase2D* program.

The subsequent figures show the visualization of the internal and external streamlines of the gas bubbles. Here *Re* is the Reynolds number based on the the bubble's diameter and rising velocity, $\kappa = \mu_G / \mu_L$ is the viscosity ratio and $\gamma = \rho_G / \rho_L$ is the density ratio, where G refers to the gas and L to the liquid phases.

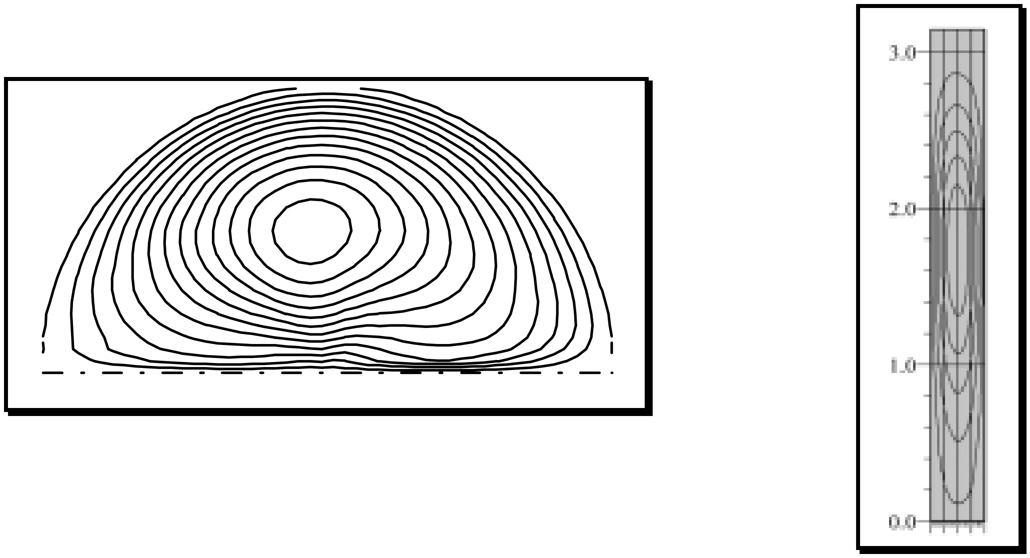

Figure 4 - Visualization of the streamlines inside a gas bubble (cylindrical and spherical coordinates): Re = 10, k = 0.022 and γ = 0.0012.

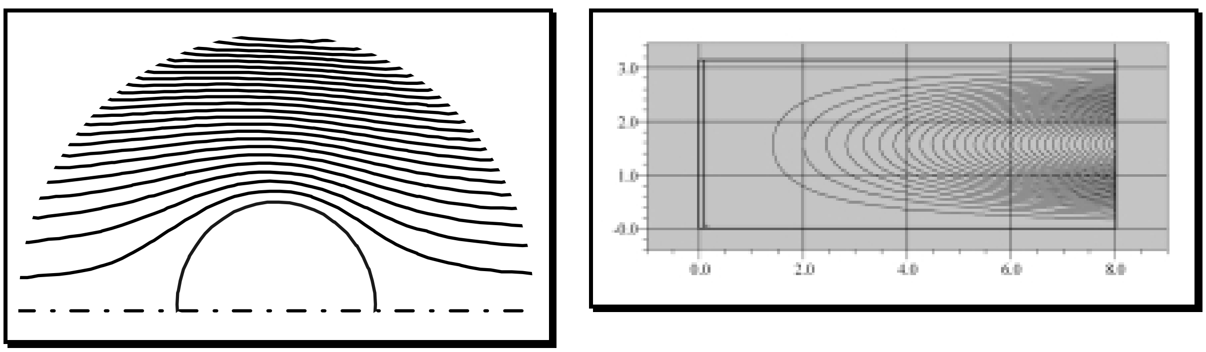

Figure 4 - Visualization of the streamlines external to a gas bubble (cylindrical and spherical coordinates): Re = 10, k = 0.022 and γ = 0.0012.

Two symmetric counter-rotating cells are observed inside the bubbles, formed, according to Silveira Neto (1998), due to a movement induction originated by viscous friction. As the movement of the external fluid increases around the bubble, a larger diffusion of momentum to the interior of the bubble occurs, so causing such recirculations.

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REFERENCES

Batchelor, G. K., 1997, An Introduction to Fluid Dynamics, Cambridge University Press, 967.

Cuenot, B., Magnaudet, J., & Spennato, B., 1997, The Effects of Slightly Soluble Surfactants on the Flow Around a Spherical Bubble, Journal of fluid Mechanics, vol. 339, pp. 25-53.

Karamanev, D. G., 1994, Rise of Bubbles in Quiescent Liquids, AIChE Journal, vol.40, nº 8 pp.1418-1421.

Magnaudet, J., Rivero, M. & Fabre, J., 1995, Accelerated flows past a rigid sphere or a spherical bubble. Part 1. Steady straining flow, Journal of Fluid Mechanics, vol.284, pp.97-135.

Reference Manual, PDEASE2D™, Macsyma Inc., 3rd. Edition, november, 1996. (http://www.macsyma.com).

Oliver, D. L. R. & Chung, J. N., 1987, Flow About a Fluid Sphere at Low to Moderate Reynolds Numbers, Journal of Fluid Mechanics, vol.177, pp.1-18.

Pantaleão, A. V., 1999, Estudo Experimental e Numérico da Ascensão de Bolhas Gasosas, Tese de mestrado, ITA, Instituto Tecnológico de Aeronáutica.

Silveira Neto, A., 1998, Simulação De Grandes Escalas de Escoamentos bifásicos Turbulentos, VII ENCIT, vol.2, pp 821 – 826, Rio de Janeiro, Brasil.